DIET: Directional Entropy based Corner Detection

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Abstract - We present a simple information theoretic approach for detecting corner points in gray level images based on a new concept of directional entropy. The gradient directions of the edge points are coded by a scheme similar to 8-directional chain codes. Based on the coded gradient directions of the edge pixels in a window, the entropy of an edge point is obtained which is referred to as the directional entropy of that edge point. Edge points with high directional entropy values are analyzed to obtain the detected corner points.

Index Terms - Corner detection, Edge detection, Entropy.

1. INTRODUCTION

Corners play an important role in computer vision and image understanding systems. Detecting corners are particularly helpful in areas like motion tracking, object recognition and stereo matching.

The present article proposes an information theoretic approach for corner detection in gray-level images. Initially gradient directions of edge points (the edge points can be obtained by any edge detection algorithm like Canny edge detection) are measured and are then mapped to distinct integers {0,1,…,7} depending on their closeness to some predefined base angles. This coding scheme is analogous to the eight directional chain codes**.**

We refer to the *directional entropy* of an edge point as the entropy of the coded gradient directions of the edge pixels in a window around that edge point. After obtaining the directional entropies of the edge points, we choose those points having high entropy values and label them as potential corners. We observe that such potential corners form various connected curve segments in and around the true corners. For each such segment, only those points having maximum directional entropy values are marked as detected corners.

The outline of the paper is as follows. Section 2 provides a brief overview of existing corner detection approaches. A few entropy measures used in our proposed method are stated in section 3. Section 4 explains the concept of directional entropy. The proposed directional entropy based corner detection approach is described in section 5. Section 6 discusses the use of some non-Shannonian definitions of entropy. The experimental results are furnished in section 7. Section 8 finds the conclusions.

2. APPROACHES TO CORNER DETECTION

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Corner detection approaches on gray level images can be divided into three categories, namely, intensity based, contour based and parametric model based approaches [1].

Intensity based methods exploit the gray level intensity information of pixels and their neighboring pixels. Moravec [2] developed one of the first corner detectors based on the autocorrelation function of the intensities. Beaudet [3] proposed a determinant operator, which has significant values only near corners. The works of Kitchen and Rosenfeld [4] included methods based on gradient magnitude of gradient direction, change of direction along edge and angle between most similar neighbors. Harris and Stephens [5] improved on the approach of Moravec by using an improved autocorrelation matrix. Smith and Brady [6] compared the brightness of each pixel in a circular mask to the center pixel to define an area which has similar brightness to the center. Trajkovic and Hedley [7] define the "cornerness" measure as the minimum change of intensity over all possible directions.

Contour based corner detection methods search for local maxima of curvature along extracted contour chains or perform some polygonal approximations and then search for intersection points. Rattarangsi and Chin [8] proposed a technique based on Gaussian scale space to detect and localize corners of planar curves. Mokhtarian and Soumela [9] use curvature scale space (CSS) to track the curvature of the image edge contours at different levels of arc length evolution. For eliminating false corners effectively, the previous method was improved by He and Yung [10] by introducing two parameters, namely, the adaptive curvature threshold coefficient C (default value of 1.5) and the corner angle, α ($160^o = \alpha = 200^o$).

Parametric model based corner detection methods usually fit a parametric intensity model to the signal. Rohr [11] used an analytic junction model, convolved with a Gaussian. Deriche and Giraudon [12] presented a general methodology for accurate identification of discrete surface intersections. Parida et al. [13] described a method for general junction detection. Lee and Bien [14] formulated the corner-detection problem as a pattern classification problem to classify pixels into two classes – corners and non-corners.

It is to be mentioned here that we have compared the performance of the proposed method with that of He-Yung method [10] as we observed it to be one of the most effective corner detection approaches.

3. ENTROPY MEASURES

Entropy [15,16] is a significant concept in information theory. The reason for the remarkable success of entropy is due to the fact that one is able to quantify uncertainty present in probabilistic systems.

Consider a probabilistic system with n states s_i; i = 1, 2, …, n. Let p_i be the probability of the i^{th} state s_i with $p_i = 0$, i = 1, 2, …, n and $\sum_{i=1}^{n} p_i = 1$. Let $P_n = (p_1, p_2, \ldots, p_n) \subset$ P, where P denotes the set of all probability distributions on finite sets. An entropy measure is a function of the form $H_n : P \rightarrow [0, \infty)$.

Several measures of entropy [15-21] have been derived so far in different contexts. These measures can be grouped into parametric and non-parametric ones. Renyi [19], Havrda and Charvat [20], and Kumar et al. [21] formulated their measures of entropy that fall into the former category. However, due to proper guidance in choosing/estimating the values of the parameters, we concentrated on the latter category of entropy measures. Among these definitions, the ones we used in this investigation are now stated.

(a) Shannon's Entropy [15]: For a discrete probability distribution P_n, Shannon, based on a set of axioms for a measure of information or uncertainty, derived the following unique definition of entropy:

$$H_{S_n}(P_n) = -\sum_{i=1}^{n} p_i \log_2 p_i . \quad (1)$$

H_{S_n} has several interesting properties such as expansibility, symmetry, continuity, monotonicity, additivity etc. A very important property for our problem is given by

$$H_{S_n}(p_1, p_2, ..., p_n) \le H_{S_n}(\frac{1}{n}, ..., \frac{1}{n}) \quad (2)$$

$\forall$ n $\in$ N, i.e., H_{S_n} attains maximum value for uniform distribution.

(b) Vajda's Quadratic Entropy [17]:

$$H_V(P_n) = \sum_{i=1}^{n} p_i (1 - p_i). \quad (3)$$

This can be viewed as an extension to Shannon's entropy. Using Taylor's series, *log(1/p_i)* can be approximated by *(1-p_i)* and H_{S_n} reduces to H_V

(c) Pal-Pal's Exponential Entropy [18]:

$$H_{PP}(P_n) = \sum_{i=1}^{n} p_i e^{(1-p_i)}. \quad (4)$$

In H_{S_n}, the gain in information from the occurrence of an event with probability p_i is taken as *log(1/p_i)*. The same for H_V is *(1-p_i)* whereas for H_{PP}, the gain function is $e^{(1-p_i)}$.

Although the previous definitions have widely different forms, they have some common characteristics. All of them satisfy expansibility, symmetry, continuity, monotonicity properties [16] and (2). Of these different properties, the most relevant one, in the present context, is that entropy attains maximum value for the uniform distribution.

4. DIRECTIONAL ENTROPY

As mentioned earlier, the proposed corner detection algorithm is based on the concept of directional entropy. The directional entropy quantifies the uncertainty of the coded gradient directions of an edge point in relation to its neighboring edge points.

In our gradient direction coding scheme, we consider eight base directions having the angles 0^o, 45^o, 90^o, 135^o, 180^o, 225^o, 270^o and 315^o, which are coded as 0, 1, 2, 3, 4, 5, 6 and 7 respectively as in Fig.1. That is, an actual gradient direction is mapped to its closest base direction. The distribution of the coded gradient directions of the edge points in a window is used in the entropy framework

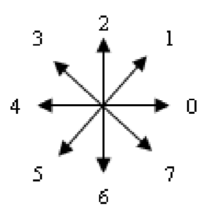

Fig.1. Base directions

Suppose $n_{w(X)}(i)$ denote the number of edge points having code *i* (*i* = 0, 1, 2, …, 7) among the total number of edge points $n_{w(X)}$ in a window *w* centered on an edge point X. The probability of having code *i* in reference to the window *w(X)*, is taken as

$$p_{w(X)}(i) = \frac{n_{w(X)}(i)}{n_{w(X)}} \tag{5}$$

Based on the probability distribution $p_i^w(X)$, *i* = 0, …, 7, the *directional entropy* for the edge point X with reference to the window *w* (centered on X) is defined using *Shannon's entropy* as

$$DE_{S_n}^{\ w}(X) = -\sum_{i=0}^{7} p_{w(X)}(i) \log_2 p_{w(X)}(i) \tag{6}$$

Note that using Shannon's definition of entropy, $DE_{S_n}^{\ w}(X) \in [0, 3]$. $DE_{S_n}^{\ w}(X)$ takes the value of zero when all the edge points in the window have the same gradient code i.e., they belong to a straight edge. On the other hand as the value of $DE_{S_n}^{\ w}(X)$ increases, the probability of the edge point X belonging to a single straight edge decreases i.e., the potentiality of the edge point to be a corner increases. Ultimately when $DE_{S_n}^{\ w}(X)$ takes the maximum value of 3 (which rarely exists in practice), all the eight gradient values are equally present in the window.

Since the directional entropy is the prime concept in finding corners, we refer to the proposed detector as DIET (**Di**rectional **E**n**t**ropy) corner detector.

5. DIET: DIRECTIONAL ENTROPY BASED CORNER DETECTION

The proposed DIET corner detection system has two main stages, namely, directional entropy computation and corner extraction. The directional entropy computation module accepts a gray level image as input and outputs the directional entropy values of the edge points. In the corner extraction module, edge points having high directional entropy values are initially retained as potential corners which are found to be situated in clusters in and around the actual corners. Finally from each of such clusters, one or more points are selected as the final detected corners.

A. Directional Entropy Computation

This module involves a sequence of few operations which include edge extraction, gradient direction computation and coding, and directional entropy computation. The concept of directional entropy of the edge points has been introduced in section 4.

There exist a good number of edge detection algorithms in the literature. Among them, Canny [22] is the most popular and has established its effectiveness in extracting edges in many image processing applications. With this consideration, we have utilized the Canny edge detector in the present investigation, although we can use any other edge detector. It has been observed that Canny edge detector produces gaps in some edge segments especially at the T-junctions. In order to overcome this shortcoming, a post edge extraction-processing operation (similar to the CSS algorithms [9,10]) is incorporated. In this step, if the endpoint of an edge contour is nearly connected to another edge contour, then the gap is filled. We further assumed that significant edge segments should be of sufficient length. For eliminating stray/insignificant edge segments, the lower threshold and the upper threshold of the Canny edge detection algorithm need to be decided accordingly. In our experiments, we have taken 0 for lower threshold and 0.35 for upper threshold.

The gradient directions of the extracted edge points are calculated using $\theta_i = \tan^{-1}(\frac{g_y}{g_x})$, where g_x *and* g_y are the horizontal and vertical components of the gradient respectively. The gradient direction θ_i (in degrees) of an edge point is mapped to the base direction code as

$$g_i = round\left(\frac{\theta_i}{45}\right) \bmod 8 \quad (7)$$

Note that $g_i \in \{0, 1, ..., 7\}$ is nothing but the closest base direction to θ_i.

Based on the coded gradient directions (θ_i's), the directional entropy $DE_{S_n}{}^w$ values are computed using (6). Recall that more the value of $DE_{S_n}{}^w$, the higher the possibility of an edge point to be a corner. This observation is utilized to extract the final corner points.

In order to illustrate the proposed directional entropy computation procedure, let us consider a typical image segment of size 8x8 as in Fig.2 having eight edge pixels with the edge vector E = {a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7}. Assuming windows of size 5x5, we obtain G = {1.591, -3.270°, 0.99°, 3.71°, 90°, 86.63°, 86.63°, 90°}; DC = {0, 0, 0, 0, 2, 2, 2, 2} and $DE_{S_n}{}^w$ = {0, 0, 0.971, 1.0, 1.0, 0.971, 0, 0}. It is to be observed here that the edge points a_3 and a_4 have the highest directional entropy value of 1.0.

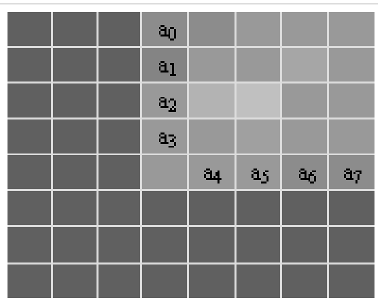

Fig. 2. A typical corner

The value of $DE_{S_n}{}^w(X)$ reflects the distribution of directional codes in the window *w*. The size of *w* should be such that it includes only the connected sequences of edge pixels. So the size of *w* may be taken to be 5x5 or 7x7 or 9x9 or 11x11 depending on the edge distribution in an image. In practice, a combined outcome of the aforementioned windows is found to be effective for any image.

B. Corner Extraction

This stage selects a few edge points (having high directional entropy values) as potential corner points and then a further refinement is applied to extract the final corner points.

Potential corner points are decided based on a threshold value of the directional entropy values. It is expected that the points in a straight edge segment have the same directional code. An intersection point of at least two straight edge segments is referred to as a corner point. If we assume that the length of the intersecting edge segments is same within the associated window, the directional entropy value (6) of the intersection or corner point becomes 1.0 (considering two segments). Under this consideration, the threshold value for selecting potential corner points is taken to be 1.0 i.e., an edge point X with $DE_{S_n}{}^{w}(X) = 1$ is marked as a potential corner point.

The potential corner set includes a few non-corner points as well, especially for step and slanted edges. Moreover, around a true corner instead of just one point, a cluster of potential corners is observed in some cases. For each cluster, we find the highest directional entropy value and then the point/points having the highest value is/are considered to be the final detected corner points.

6. USE OF NON-SHANNONIAN ENTROPY DEFINITIONS

So far, the proposed algorithm dealt with only the Shannon's entropy measure. The effectiveness of two other non-Shannonian entropy definitions, namely, Vajda's (3) and Pal-Pal's (4), is now investigated. As the range of directional entropy differs with the entropy measures, different threshold values for extracting potential corners need to be considered accordingly.

Vajda's Quadratic Entropy [17] : The directional entropy for an edge point X in reference to the window *w* (centered on X) is defined using Vajda's definition as

$$DE_V{}^{w}(X) = \sum_{i=0}^{7} p_{w(X)}(i)(1 - p_{w(X)}(i)) \tag{8}$$

where $DE_V{}^{w} \in [0, 0.875]$. The threshold value of the directional entropy (with this definition) for extracting potential corners is 0.5.

Applying (8) on the edge points $\{a_0, a_1, \ldots, a_7\}$ in Fig. 2, we obtain (using 5x5 windows), $DE_V{}^{w} = \{0, 0, 0.48, 0.5, 0.5, 0.48, 0, 0\}$. As expected, the edge points a_3 and a_4 have the highest values of directional entropies, and they are marked as potential corners.

Pal-Pal's Exponential Entropy [18] : The directional entropy for an edge point X in reference to the window *w* (centered on X), the directional entropy using Pal-Pal's definition is obtained as

$$DE_{PP}{}^{w}(X) = -\sum_{i=0}^{7} p_{w(X)}(i)e^{(1-p_{w(X)}(i))} \tag{9}$$

Note that $DE_{PP}{}^{w} \in [0, 2.3989]$. The threshold for selecting potential corners using this definition is taken to be 1.6489.

We find, $DE_{PP}{}^{w} = \{0, 0, 1.6239, 1.6489, 1.6489, 1.6239, 0, 0\}$ corresponding to the edge points $\{a_0, a_1, \ldots, a_7\}$ in Fig. 2. Here too, edge points a_3 and a_4 are marked as potential corners.

It is to be noted here that the lower bound for $DE^{w}(X)$ is 0.0 (using any entropy measure) which happens when the points lie on a straight edge. On the other hand, the theoretical highest value of directional entropy with any measure represents a highly ambiguous scenario and hardly occurs in practice.

7. EXPERIMENTAL RESULTS

The effectiveness of the proposed corner detection system has been verified with some artificially generated images as well as with many real-world images. Here we have demonstrated the results on two real images, namely, the blocks image and the house image. Note that both these two standard images are of size 256x256 pixels and have been used in almost all corner detection approaches.

The original blocks image with the actual (reference) corner points is shown in Fig. 3(a). The edges obtained are provided in Fig. 3(b). The final output of our system i.e., the detected corner points using directional entropy definitions with Shannon, Vajda and Pal-Pal's measures are provided in figures 3 (c)-(e) respectively. Similarly the original house image, the edges detected and corner points extracted by the proposed algorithm using Shannon, Vajda and Pal-Pal's measure of entropy are shown in figures 4 (a)-(e).

For comparison, the detected corner points obtained by the He-Yung method [10] (using the parameter values of C and α as 1.5 and 160^o respectively as suggested by the authors) are shown in figures 3(f) and 4(f) for the blocks image and the house image respectively. It may be noted here that the performance of our method is quite comparable with that of [10].

Evaluation results for the blocks and house images by both our method and [10] are presented in Table 1. Here we consider a detected corner to be 'correct' if the corresponding reference point is within a distance of 4 pixels as in [10]. The remaining corner points in the detected corner set and reference set are labeled as 'false' and 'missed' corners accordingly. As far as detection of the correct corners is concerned, our results are much better than that of [10]. All the three entropy definitions provide comparable results for the blocks image whereas the entropy definitions of Vajda and Pal-Pal are more effective than Shannon in the house image. For both the images, the proposed method is seen to produce much more false corners.

Table 1. Performance evaluation on the blocks and house images.

Images	Corner Detectors	Correct corners	Missed Corners	False Corners
Blocks	**DIET using**			
	Shannon	51	8	9
	Vajda	52	7	17
	Pal+Pal	51	8	14
	He-Yung	42	17	2
House	**DIET using**			
	Shannon	44	24	20
	Vajda	54	14	23
	Pal+Pal	56	12	23
	He-Yung	44	24	10

8. CONCLUSION

This article introduced a new concept of directional entropy and accordingly proposes a new method for corner detection. Indeed this work demonstrated the potential of entropy in low level image processing. Our proposed method is based on the hypothesis that the edge points around corners will have higher directional entropy values than those on straight edges. Experimental results reveal that the proposed method is quite effective in identifying the corners in gray level images. We observe that most of the 'false' corners exist in the slant/step edges, which can be overcome to some extent by using a suitable edge smoothing operation. Minute inspection of each of the missed corners by the proposed method reveals that they are due to improper/non- extraction of corresponding edges or rounded shape of corners (lack of significant sharpness of corners with the associated edges) or presence of multiple corners in very near proximity.

The efficacy of the method is dependent on the output of the edge detector used (Canny). The application of directional entropy for edge detection may also be explored which can reduce the dependency on the external edge detector. As the method is based on codes instead of actual gradient directions, the system tends to be effective under rotational and affine transformations. Results of such investigations will be reported in our future communications.

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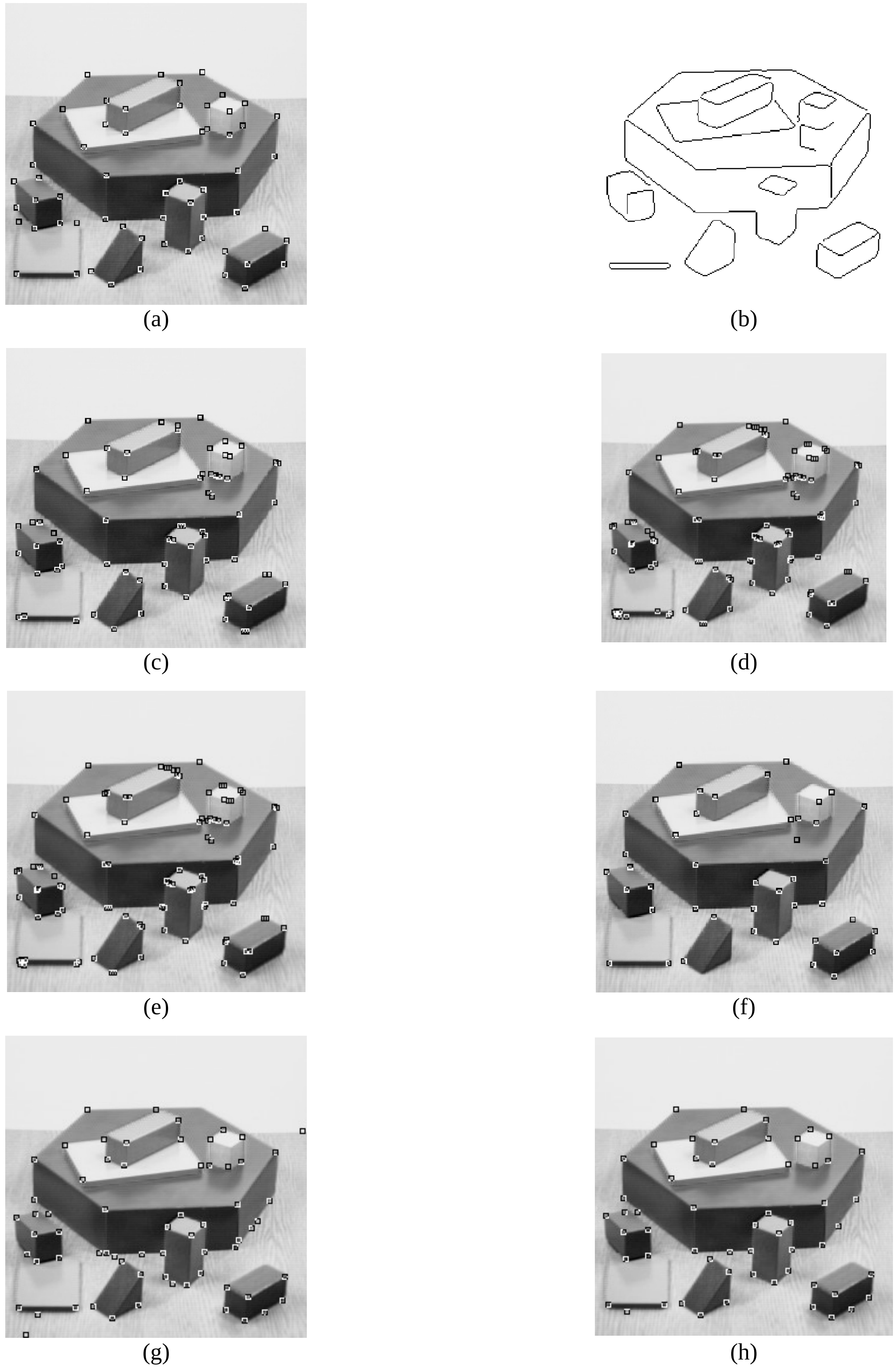
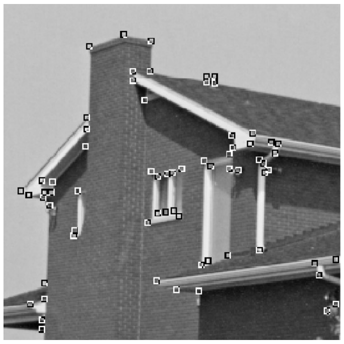


Fig. 3. Results on blocks image: (a) Original image with actual corners, (b) Edges, (c) Corners detected by DIET using Shannon's entropy, (d) Corners detected by DIET using Vajda's entropy, (e) Corners detected by DIET using Pal-Pal's entropy, (f) Corners detected by He-Yung [10], (g) Corners detected by Harris [5] (σ = 1.45, t = 850, r = 3), (h) Corners detected by Harris [5] (σ = 1.45, t = 850, r = 3).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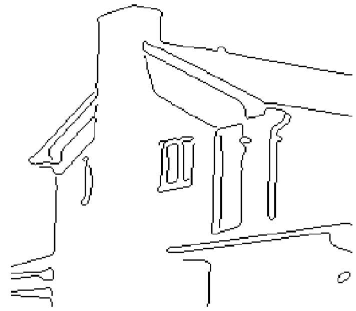
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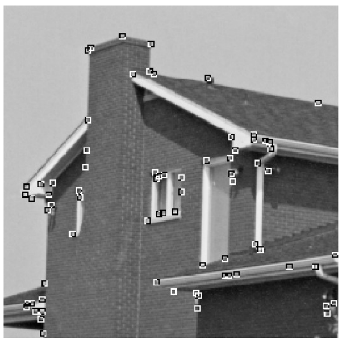
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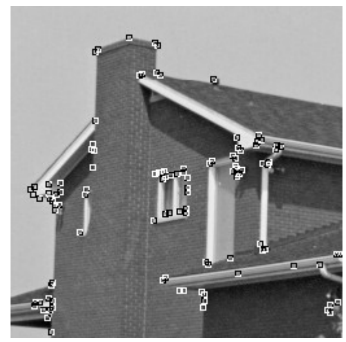
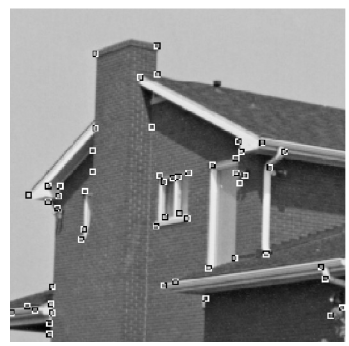

(d)


(e)


(f)


(g)


(h)

Fig. 4. Results on house image: (a) Original image with actual corners, (b) Edges, (c) Corners detected by DIET using Shannon's entropy, (d) Corners detected by DIET using Vajda's entropy, (e) Corners detected by DIET using Pal-Pal's entropy, (f) Corners detected by He-Yung [10], (g) Corners detected by Harris [5] (σ = 1.45, t = 850, r = 3), (h) Corners detected by Harris [5] (σ = 1.45, t = 850, r = 3).